

Binary coalescence of a strange star with a black hole: Newtonian results.

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Abstract

We present Newtonian three-dimensional hydrodynamical simulations of the merger of quark stars with black holes. The initial conditions correspond to non-spinning stars in Keplerian orbits, the code includes gravitational radiation reaction in the quadrupole approximation for point masses. We find that the quark star is disrupted, forming transient accretion structures around the black hole, but 0.03 of the original stellar mass survives the initial encounter and remains in an elongated orbit as a rapidly rotating quark starlet, in all cases. No resolvable amount of mass is dynamically ejected during the encounters—the black hole eventually accretes $99.99\% \pm 0.01\%$ of the quark matter initially present.

1 Introduction

In this paper we study the binary coalescence of a black hole and a quark star. Stellar population studies indicate that if quark stars and black holes exist at all, such binaries should exist in numbers significant from the point of view of next-generation laser interferometric gravitational

wave detectors, but smaller than the number of Hulse-Taylor type binaries (Belczyński et al. 2001). Coalescing quark stars also remain strong candidates for gamma-ray burst sources (Paczynski 1991, 2001; Haensel et al. 1991). This is reason enough to study such coalescences. However, in this numerical study we address only one specific question: how much quark matter, and with what velocities, is ejected when a quark star coalesces with a black hole. The interest here is in the speculations that such ejecta may convert all neutron stars to quark stars.

2 Quark stars, neutron stars, and the contamination problem

Bodmer (1971) and Witten (1984) have conjectured that a three-flavor quark fluid is the ground state of hadronic matter. This (up, down and) strange quark matter, if cold, would be stable in the bulk at zero pressure—spontaneous fission into individual hadrons or (hyper) nuclei would only occur for the tiniest specks of quark matter, composed of less than a few thousand quarks each (Farhi and Jaffe, 1984). As Witten (1984) was quick to point out, a sufficiently large (self-gravitating) quantity of quark matter would be remarkably similar to conventional neutron stars—a solar mass quark star would be about 10 km across, and the maximum mass of a quark star stable against collapse to a black hole would be about $2M_{\odot}$. The TOV equation for quark stars of masses up to the maximum value has also been solved by Itoh (1970); Brecher and Caporaso (1976); Haensel, Zdunik and Schaeffer (1986); as well as by Alcock, Farhi and Olinto (1986a), who also give a detailed discussion of the properties of these “strange” stars and of the astrophysical context. For recent reviews see Cheng, Dai and Lu (1998) and Madsen (1999).

The existence of self-bound quark matter and of quark stars remains a hypothesis. Even so, we are now witnessing a revival in their theoretical study, prompted no doubt by advances in computational techniques and in the maturing of X-ray and gamma-ray astronomy, as well as by a deeper understanding of collective effects in quark matter (e.g., Alford, Rajagopal and Wilczek 1998; Rapp et al. 1998).

Several groups have computed the structure of rotating quark stars in full general relativity, and discussed their external metric in the context of kHz quasi-periodic oscillations (QPOs) observed in certain X-ray binaries (Gourgoulhon et al. 1999, Stergioulas et al. 1999, Gondek-Rosińska et al. 2001, Bombaci et al. 2000), and an even greater number have investigated the possible connection between quark stars and energetic phenomena such as gamma-ray bursts (Paczynski 1991; Haensel, Paczynski and Amsterdamski 1991; Cheng and Dai 1996), soft gamma-repeaters, a.k.a. SGRs (Alcock, Farhi and Olinto 1986b; Horvath et al. 1993; Cheng and Dai 1998, 2002; Dai and Lu 1998; Zhang, Xu and Qiao 2000; Usov 2001), and radio pulsars (Xu et al. 1999). This reference list is far from exhaustive.

However, there is a shadow over all this activity. The Galaxy must not be contaminated with “seeds” of quark matter. Such seeds present no danger to the Earth, because ions are repelled by the high (~ 10 MeV) Coulomb barrier surrounding quark matter of the type discussed here (Farhi and Jaffe 1984; Alcock et al. 1986a). But neutrons are easily absorbed and dissolved by quark matter, and Witten (1984) noted that if a coalescing binary of the Hulse-Taylor type contains a quark star, and if Clark and Eardley’s (1977) speculation on the amount of matter ejected (said to be $\sim 0.1M_\odot$) is correct, then “there could be $10^5 M_\odot$ of quark matter free in our Galaxy” and even the smallest bit of quark matter inside a neutron star would convert it to a quark star by absorbing all neutrons. On the other hand, it has been argued that some radio pulsars are neutron stars and cannot be quark stars (Alpar 1987, Madsen 2000). This would place an extremely severe limit on the space density of quark nuggets, a limit thought to be drastically violated by the expected ejection of matter in those coalescing binaries in which at least one of the components is a quark star, and the other is equally or more compact (Madsen 1988, Caldwell and Friedman 1991).

3 Aim of this study

We would like to find out if binary coalescence events do indeed pose a problem for the hypothetical co-existence of quark stars and neutron stars in our Galaxy. There are several aspects to this issue.

The first and non-controversial statement is that if quark matter is stable, then even the smallest quark seed present in the interior of a neutron star would convert it to a quark star (Witten 1984, Olinto 1987). Such a seed may arise spontaneously when neutron-star matter exceeds a certain density, as in the center of neutron stars or in supernovae, or it may be captured from the ambient medium by a neutron star or its massive stellar progenitor (Alcock et al. 1986a). It is the latter possibility that concerns us here.

The second question is whether neutron stars and quark stars actually co-exist in our Galaxy. If quark fluid is not the most stable form of hadronic matter, the issue is moot. Ditto, if all the presumed neutron stars (radio pulsars, X-ray bursters, etc.) are in fact quark stars. The question really arises only if at least some of the observed compact objects are neutron stars. It has been argued that glitching radio pulsars (such as the Crab and Vela) are really conventional neutron stars (Alpar 1987)—the observed occasional impulsive changes in the period of such young radio pulsars have been understood in terms of redistribution of angular momentum at the base of the crust of neutron stars, and quark stars are unable to support a crust of the requisite moment of inertia (Alcock et al. 1986a). It has also been argued that the r-mode instability limits the rotation rate of quark stars, at least for some forms of quark matter, to values far below the ones observed in millisecond pulsars (Madsen 2000). These arguments would lead to very stringent limits on the space density

of quark matter, although it has also been argued that recent work on the crystalline phase of superfluid quark matter may alleviate these concerns (Alford et al. 2000).

The third and fourth questions relate to the total mass of quark matter dispersed in the Galaxy and to the size distribution of its fragments. It is generally *assumed* that about $0.1M_{\odot}$ of quark matter is ejected in a coalescing quark star binary. Madsen (1988) finds that if the ejected matter is dispersed into small nuggets of baryon number $A < 10^{28}$, i.e., less than (possibly much less than) 10 kg in mass, then such a nugget can be captured by a pre-supernova star and come to rest at the center of its core. As discussed above, this would lead to the conversion of any neutron star, subsequently formed in the supernova, to a quark star. Madsen then concludes that even one coalescence event ejecting quark matter would be enough to seed all pre-supernova stars. Caldwell and Friedman (1991) specifically discuss the fate of quark matter fragments formed in the disruption of a quark star coalescing with a black hole, and come to the same conclusions.

The disrupted star has a mass exceeding 10^{30} kg, i.e., $A \sim 10^{57}$. No numerical simulation is currently capable of having a dynamic range of 29 orders of magnitude, so it is unrealistic to expect the hydrodynamic computation reported here to settle the issue of the size of the droplets into which quark matter breaks up in the coalescing binary. But it is possible, and this is the main aim of the present study, to set upper limits to the total amount of quark matter ejected in the form of droplets. We have previously successfully performed simulations of coalescing “neutron star” binaries allowing a mass resolution down to $10^{-5}M_{\odot}$ to be achieved on desk-top work-stations (Lee and Kluźniak 1995; Kluźniak and Lee 1998; and papers I,II,III,IV: Lee and Kluźniak 1999a,b, Lee 2000, 2001, respectively). This may not seem to be very constraining, as Caldwell and Friedman (1991) argue that after 10^6 coalescences of strange–black hole systems “the average strangelet content exceeds by forty-one orders of magnitude the minimum density needed to seed the star.” But in fact, the difference between between $10^{-1}M_{\odot}$ and $10^{-3}M_{\odot}$ may be critical, because forty-four of the forty-one orders of magnitude come from the assumed fragmentation of quark matter, supposed to be occurring close to the black hole. In our Newtonian simulations only a small amount of matter ($\sim 10^{-2}M_{\odot}$) never approaches the black hole, and if it is a fraction of this matter that is ejected, the fragmentation into small quark nuggets may never have occurred.

If it is true, as we find, that Clark and Eardley’s guess of $0.1M_{\odot}$ for the mass ejected in the coalescence of two *neutron* stars—a value uncritically adopted (also by Kluźniak, 1994) in all estimates of the ambient density of quark nuggets in the Galaxy—is not supported by actual hydrodynamic simulations of the coalescence of a *quark* star and a black hole, astrophysical arguments against the existence of stable quark matter at zero pressure will have to be re-examined.

Much of the argument for dispersal of quark nuggets in the Galaxy

rests on the assumption of violent collisions between quark fragments, prior to their ejection from the binary. A subsidiary aim of this paper is to check whether the coalescence process is indeed conducive to fragmentation of quark matter into fragments of low baryon number.

4 The choice of simulation

In this paper we report the results of a Newtonian SPH study of coalescing binaries in which one component is a massive quark star and the other a black hole, about twice or three times as massive. To our knowledge, this is the first 3-d hydrodynamical study to be performed of coalescing quark stars. We have chosen to first study a black hole as the second component in the binary rather than a quark star, for four reasons, giving us some measure of confidence that the results reported may be qualitatively correct. First, the coalescence process is essentially over much more quickly when one of the components is a black hole, allowing more simulations to be carried out in a given time. This is both because the actual physical process is shorter and because the number of SPH particles decreases during the simulation as they are swallowed by the black hole. Second, all the SPH particles in this simulation model a single quark star, instead of two. This gives a substantial gain in the resolution. Third, in the Newtonian approach used here, it is easier to simulate some qualitative features of general relativity in black-hole accretion than in accretion onto material stars. The only qualitative effect of general relativity modeled in the present study is the irreversible accretion of matter by the black hole—the Paczyński-Wiita potential has been widely used in other studies of accretion disk to mock up the presence of the marginally stable orbit, but the simulations reported here relied on a fully Newtonian potential. Fourth, unlike in the case of two coalescing non-rotating stars, no vortex sheet is expected to form in a black hole binary.

5 Numerical Method

For the calculations presented in this paper, we have used the method known as Smooth Particle Hydrodynamics (SPH) (see Monaghan 1992 for a review of the method). The code is essentially the same one used previously to model the coalescence of black holes with polytropic stars (papers I–IV). The equations of motion include an artificial viscosity term, to handle the presence of shocks and avoid particle interpenetration. The standard form (see Monaghan 1992) includes terms both for shear and bulk viscosity. During dynamical simulations of coalescing binaries, accretion disks are often formed, and thus the effects of a shear viscosity can have a substantial impact on their evolution. To minimize this effect, we have used the artificial viscosity prescription proposed by Balsara (1995). The momentum and energy equations for a given SPH particle are then

given by:

$$\frac{d\vec{v}_i}{dt} = - \sum_j m_j \left(\frac{2\sqrt{P_i P_j}}{\rho_i \rho_j} + \Pi_{ij} \right) \nabla_i W_{ij} - \nabla \Phi_i + \vec{a}_i^{RR}, \quad (1)$$

and

$$\frac{du_i}{dt} = \frac{1}{2} \sum_j m_j \left(\frac{2\sqrt{P_i P_j}}{\rho_i \rho_j} + \Pi_{ij} \right) (\vec{v}_i - \vec{v}_j) \cdot \nabla_i W_{ij}. \quad (2)$$

Here Π_{ij} is the artificial viscosity term and \vec{v} , P , u , Φ , a^{RR} and W are the velocity, pressure, internal energy per unit mass, gravitational potential (which includes the self gravity of the fluid as well as the contribution arising from the presence of the black hole), gravitational radiation reaction acceleration, and the smoothing kernel respectively. For the kernel we use the spline form of Monaghan & Lattanzio (1985). The viscous term is given by

$$\Pi_{ij} = \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) (-\alpha\mu + \beta\mu_{ij}^2) \quad (3)$$

where

$$\mu_{ij} = \begin{cases} \frac{(\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)}{h_{ij}(|\mathbf{r}_i - \mathbf{r}_j|^2/h_{ij}^2) + \eta^2} \frac{f_i + f_j}{2c_{ij}}, & (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) < 0 \\ 0, & (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) \geq 0 \end{cases}$$

and f_i is the form-function for particle i defined by

$$f_i = \frac{|\nabla \cdot \mathbf{v}|_i}{|\nabla \cdot \mathbf{v}|_i + |\nabla \times \mathbf{v}|_i + \eta' c_i / h_i}.$$

The factor $\eta' \simeq 10^{-4}$ in the denominator prevents numerical divergences. The sound speed at the location of particle i is denoted by c_i , and α and β are constants of order unity. The divergence and curl of the velocity field are evaluated through

$$(\nabla \cdot \mathbf{v})_i = \frac{1}{\rho_i} \sum_j m_j (\mathbf{v}_j - \mathbf{v}_i) \cdot \nabla_i W_{ij}$$

and

$$(\nabla \times \mathbf{v})_i = \frac{1}{\rho_i} \sum_j m_j (\mathbf{v}_j - \mathbf{v}_i) \times \nabla_i W_{ij}.$$

This form of the viscosity vanishes in regions of strong vorticity, when $\nabla \times \vec{v} \gg \nabla \cdot \vec{v}$, but remains in effect if compression dominates in the flow ($\nabla \cdot \vec{v} \gg \nabla \times \vec{v}$). This allows us to minimize the effects of artificial viscosity on the evolution of disk-like structures in the simulations, when they appear.

In this study, unlike before, we take the gravitational acceleration of a volume of fluid to be proportional to its total energy density, i.e., we

reinterpret ρ in all the above equations (but not in the self-gravity term implicit in Φ_i) as the energy density divided by c^2 , and we add any changes in the internal energy (eq. [2]) to ρc^2 . To model quark matter we use the simplest MIT equation of state (e.o.s.), where the pressure is given by $P = c^2(\rho - \rho_0)/3$ for $\rho > \rho_0$, and is zero otherwise. Note that for $\rho \leq \rho_0$ the viscous stress vanishes (eq. [3]), and when the radiation reaction is turned off as well the equation of motion (1) is that of dust.

In compact binaries, the orbital decay is driven primarily by the emission of gravitational waves. To take this effect into account, we include the back reaction on the system, computed in the quadrupole approximation for point masses (see e.g. Landau & Lifshitz 1975), so that the rates of energy and angular momentum loss are given respectively by

$$\frac{dE}{dt} = -\frac{32}{5} \frac{G^4 (M_{SS} + M_{BH}) (M_{SS} M_{BH})^2}{(cr)^5} \quad (4)$$

and

$$\frac{dJ}{dt} = -\frac{32}{5c^5} \frac{G^{7/2}}{r^{7/2}} M_{BH}^2 M_{SS}^2 \sqrt{M_{BH} + M_{SS}}, \quad (5)$$

where r is the binary separation (defined as the distance between the black hole and the center of mass of the strange star).

The corresponding acceleration on each binary component is then given by:

$$\mathbf{a}^* = -\frac{1}{q(M_{SS} + M_{BH})} \frac{dE}{dt} \frac{\mathbf{v}^*}{(v^*)^2} \quad (6)$$

$$\mathbf{a}^{BH} = -\frac{q}{M_{SS} + M_{BH}} \frac{dE}{dt} \frac{\mathbf{v}^{BH}}{(v^{BH})^2} \quad (7)$$

where v^* is the velocity of the quark star and v^{BH} that of the black hole, q is the mass ratio of the components.

The application of the above equations is trivial in the case of the black hole, since we always treat it as a point mass. For the star, we apply the same acceleration to each SPH fluid particle, using equation (6) evaluated at the center of mass of the fluid, so that we have:

$$\mathbf{a}^i = -\frac{1}{q(M_{SS} + M_{BH})} \frac{dE}{dt} \frac{\mathbf{v}_{cm}^*}{(v_{cm}^*)^2}. \quad (8)$$

Once the star is tidally disrupted, this approximation clearly becomes meaningless, and so we switch off the corresponding terms when the binary separation becomes smaller than the tidal disruption radius $r_{tidal} = CR_{SS}(M_{BH}/M_{SS})^{1/3}$, where C is a constant of order unity. This formulation of gravitational radiation back-reaction has been used before for coalescing compact binaries (e.g. Davies et al. 1994; Rosswog et al. 1999; Lee & Kluźniak 1999b).

Table 1: Initial conditions ($N = 17256$ for all runs)

Run	ρ_0 [$10^{14} \text{ g cm}^{-3}$]	M_{SS} [M_\odot]	R_{SS} [km]	t_{ff} [ms]	q	r_i [R_{SS}]	ν_{orb} [Hz]
A	7.318	1.5	9.0	0.06	0.5	3.25	775.59
B	7.318	1.5	9.0	0.06	0.3	3.70	767.37
C	4.116	2.0	12.0	0.08	0.5	3.25	581.69
D	4.116	2.0	12.0	0.08	0.3	3.70	575.53
E	2.634	2.5	15.0	0.10	0.5	3.25	465.35
F	2.634	2.5	15.0	0.10	0.3	3.70	460.42

6 Initial conditions

We initially construct a spherical star by placing N particles of equal mass on a cubic three-dimensional grid and including a damping term in the equations of motion for an isolated star. The system then relaxes for approximately twenty freefall times ($t_{ff} \approx (G\bar{\rho})^{-1/2}$). Table 1 shows the initial parameters used for our dynamical runs. We have used three different values for the initial mass of the strange star, corresponding to the maximum mass for a given value of ρ_0 (as noted by Witten [1984], $M_{\max} \propto \rho_0^{-1/2}$; for a discussion of physical bounds on M_{\max} see Zdunik et al. [2000]). The black hole is modeled as a spherical vacuum cleaner—a point mass producing a Newtonian potential $\Phi = -GM_{BH}/r$, with an absorbing boundary at the Schwarzschild radius $r_{Sch} = 2GM_{BH}/c^2$. The mass ratio is defined as $q = M_{SS}/M_{BH}$. For each value of M_{SS} we have performed calculations for two different values of q , giving a total of six dynamical runs, shown in Table 1.

To perform the dynamical simulations (described below), we place the star a distance r_i from the black hole and give the binary components the azimuthal velocity corresponding to a Keplerian binary with angular velocity $2\pi\nu_{orb} = \sqrt{G(M_{SS} + M_{BH})/r^3}$, plus the radial velocity corresponding to point-mass inspiral.

Every SPH particle in the star is given the same azimuthal velocity, and thus the system corresponds to one in which the star is not spinning in an external (inertial) frame of reference. We have two reasons for choosing a non-spinning star at the beginning of the run. This initial condition is believed to be realistic since the shear viscosity of quark matter is believed to be smaller than in neutron-star matter, and in neutron stars tidal synchronization can be neglected (Kochanek 1992; Bildsten & Cutler 1992). Further, past experience (papers I through IV) teaches us that the ejection of matter from tidally locked polytropes is much smaller than from non-spinning polytropes (for which the coalescence process is much more violent). The present simulations of quark-star coalescence resemble to a certain extent our earlier simulations for stiff polytropes, hence we expect the same dependence to hold here. A non-spinning quark star is

the right choice for the initial conditions, if we are to place secure upper bounds on the amount of matter ejected from the binary.

As it turns out, at the start of the dynamical calculation, a tidal bulge appears on the star, in the direction facing the black hole. This is simply because the initial configuration (i.e., spherical star plus point-mass companion) is not in equilibrium at $t = 0$. Thereafter the star spirals in due to gravitational radiation reaction (at the initial binary separations given in Table 1, the decay timescale due to the emission of gravitational waves is comparable to the orbital period). In trial runs we have placed the star also at various larger initial separations. The outcome of the coalescence was found to be insensitive to the choice of r_i .

7 Comparison with previous simulations

We are in a position to compare the results presented here for quark-matter e.o.s., $P = c^2(\rho - \rho_0)/3$, with those obtained for the polytropic e.o.s. $P = K\rho_b^\Gamma$ or $P = (\Gamma - 1)\rho_b u$ (here ρ_b denotes the baryon rest-mass density). We had previously carried out coalescence simulations, with the same code as the one used here, for stiff polytropes with $\Gamma = 3$ (papers I, III), and for soft polytropes with $\Gamma = 5/3$ and $\Gamma = 2$ (papers II, IV). In all cases, the second component was taken to be a black hole. As remarked in Section 6, we have found that the coalescence for tidally locked binaries (papers I, II) was less violent than that for initially non-spinning polytropes (papers III, IV)—for a stiff polytrope and a binary with $q < 1$, the tidally locked polytrope dribbled mass at discrete intervals, while the irrotational polytrope was almost completely tidally disrupted in a single episode of mass transfer.

Since the mass relationship for a polytrope is $d \log R / d \log M = (\Gamma - 2)/(3\Gamma - 4)$, a stiff polytrope ($\Gamma > 2$) responds to mass loss by shrinking, while the soft ones ($\Gamma < 2$) expand when losing mass. In this sense, the soft polytropes (which were always completely disrupted in their first approach to the black hole) are a better model for neutron stars (for which $dR/dM < 0$, e.g., Arnett and Bowers 1977), while the stiff ones may be taken as an approximation to quark stars. Quark matter is nearly incompressible, with a density contrast less than a factor of five inside a quark star (Witten, 1984), and for lower mass quark stars $M \propto R^3$ is a good approximation (Alcock et al., 1986a). In general, the volume of quark stars increases with their mass, and for $P \propto \rho - \rho_0$ the effective polytropic index goes to infinity as $\rho \rightarrow \rho_0$ (Haensel et al. 1986), so the outer parts of the (bare) quark star behave like an extremely stiff polytrope. We expected and found that the overall evolution in the coalescence of a quark star is very similar to that observed for black hole binaries with a stiff polytrope (paper III). However, there are important differences, notably in the amount of matter ejected.

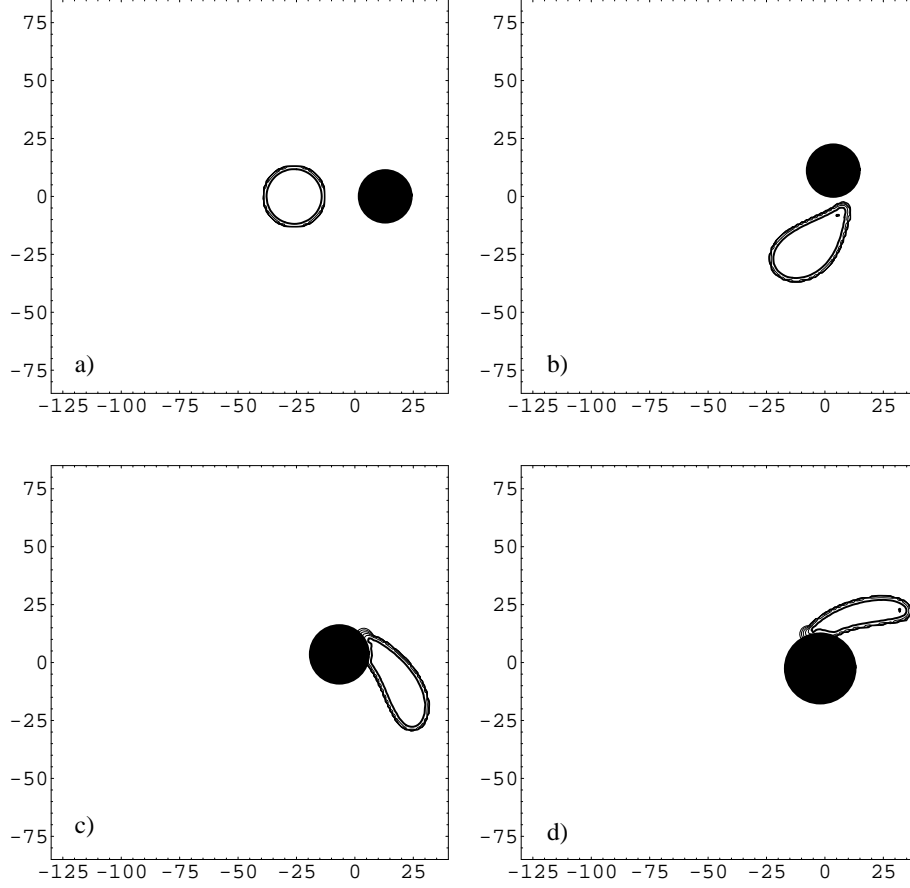


Figure 1: Density contours in the orbital plane at (a) $t = 0$, (b) $t = 0.32$ ms, (c) $t = 0.64$ ms, (d) $t = 0.97$ ms, (e) $t = 1.29$ ms, (f) $t = 1.61$ ms, (g) $t = 1.94$ ms, and (h) $t = 2.26$ ms, for run C ($M_{\text{SS}} = 2.0M_{\odot}$, $R_{\text{SS}} = 12.0$ km). Orbital rotation is counterclockwise, axes are labeled in km. Contours are equally spaced every 0.5 dex, starting at $1.3 \times 10^{12} \text{ g cm}^{-3}$, with the highest contour in bold at $\rho = \rho_0 \equiv 4.116 \times 10^{14} \text{ g cm}^{-3}$. Initially, as in panel a), the contours outside this bold contour are a numerical artifact reflecting the size of the SPH kernel on the surface of the quark star. At later stages, the thin contours represent average density of quark “dust” composed of particles of unresolved mass, less than $64M_{\text{SS}}/17256$.

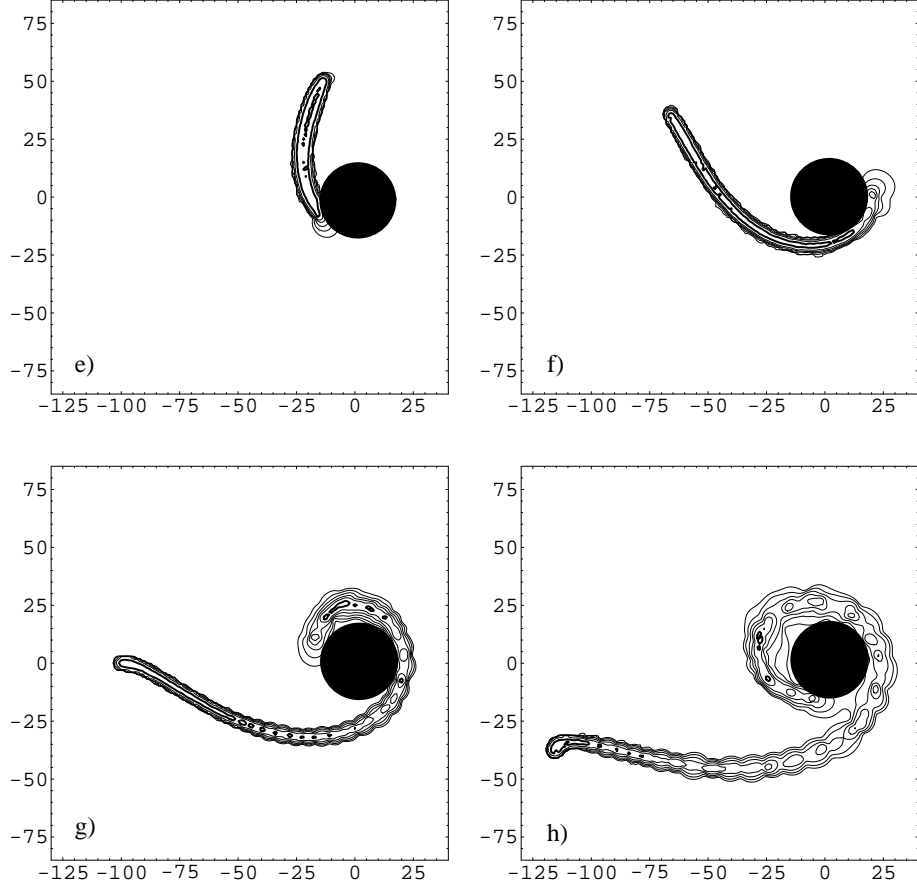


Figure 1: continued. Note the appearance of a large “vacuum” region of density below ρ_0 in the middle of the very elongated quark star in panel e) (bold contour within a bold contour), and smaller such regions in panels b), d), and f). In panels g) and h), in addition to dust, several blobs of quark fluid on their way to the black hole are clearly visible. One such blob is seen to be separating from the tubular quark star in panel f).

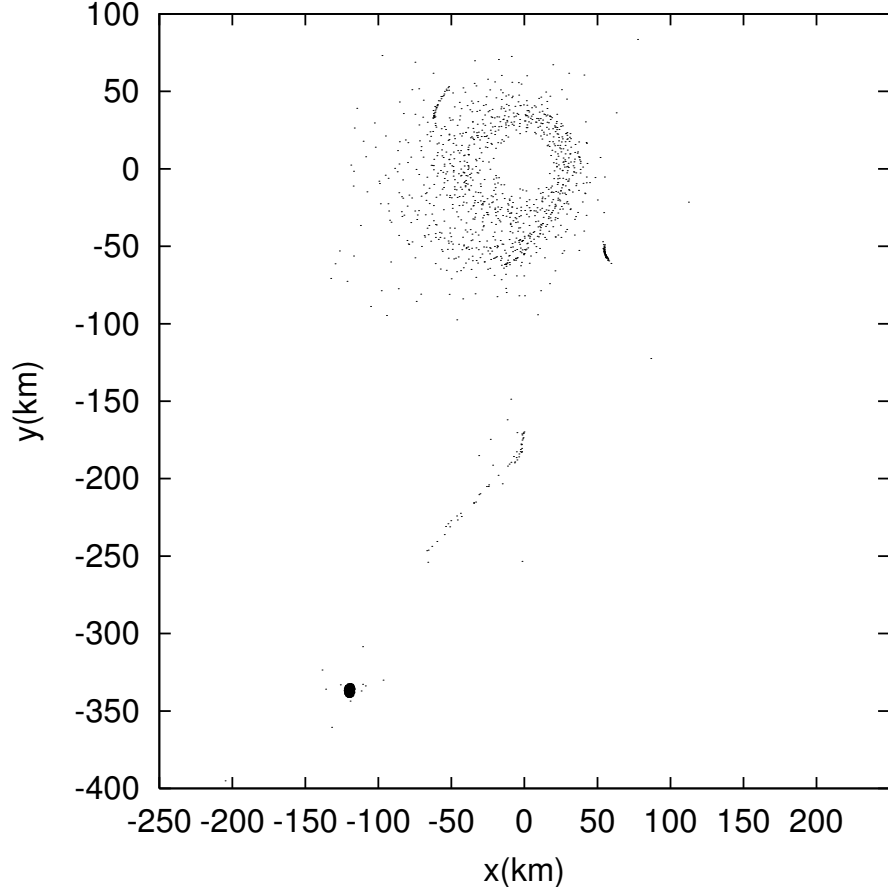


Figure 2: SPH particle positions at the end of the calculation for run C (at $t = 6.46$ ms), projected onto the orbital plane. With the exception of the clearly visible clump at the end of the disrupted tidal tail, the as-yet-not-accreted quark matter has dispersed into droplets of unresolved mass. All SPH particles (dots in this figure) have the same mass, $m_i = M_{\text{SS}}/N$, so the mass density in the zero-pressure “fog” of these droplets is proportional to the number of SPH particles per unit volume. Everything visible in the figure, save at most two particles, will eventually be accreted by the black hole.

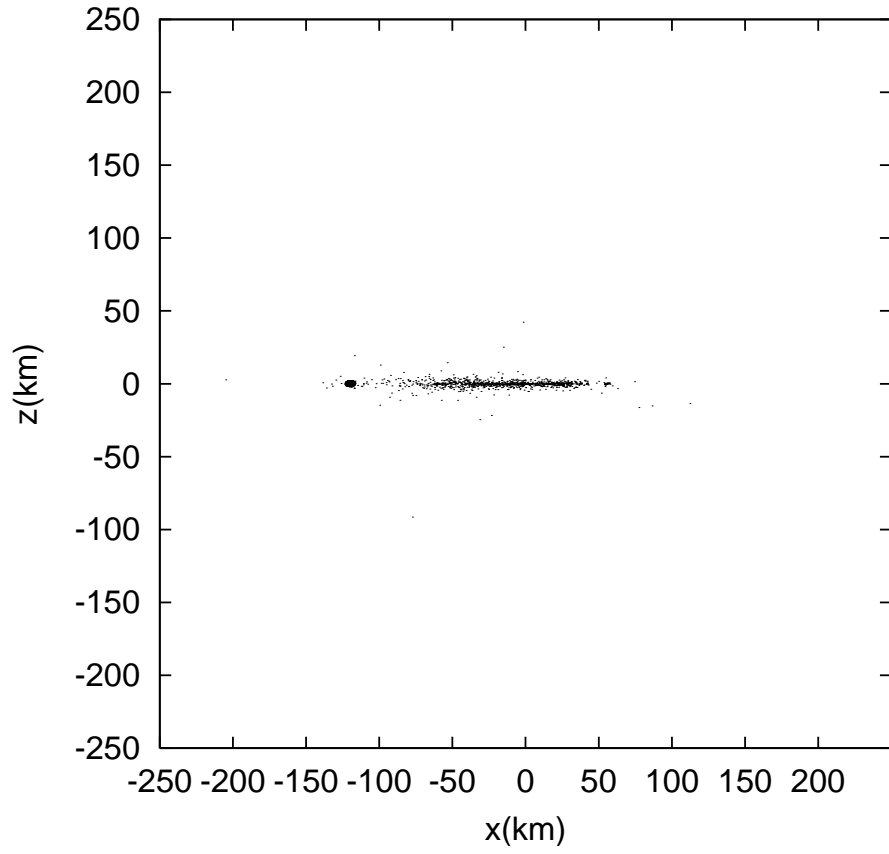


Figure 3: SPH particle positions at the end of the calculation for run C (at $t = 6.46$ ms), projected onto the meridional plane $y = 0$.

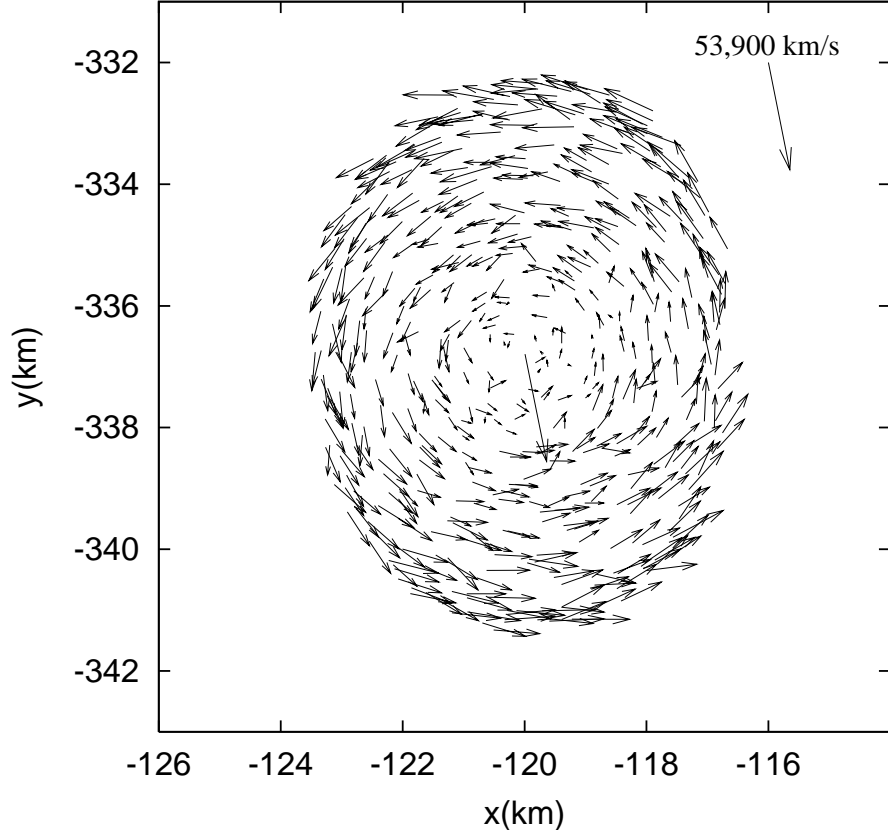


Figure 4: Relative velocity field for the clump at the end of run C, projected onto the orbital plane (compare Fig. 2). The arrow in the top-right corner shows the clump center-of-mass velocity vector (which has been subtracted from individual particle velocities to show the rotational motion). The mass of this volume of quark fluid orbiting the black hole is $0.055M_{\odot}$, and the inferred angular velocity of rotation is $\Omega_{\text{rot}} \approx 5 \times 10^3 \text{ s}^{-1}$.

8 Results

All our computations (regardless of the initial mass M_{SS} and mass ratio q) give qualitatively similar results. The star becomes quickly elongated due to tidal forces, initiating mass transfer onto the black hole within one orbital period. As the accretion stream winds around the black hole, it forms a disk-like structure around it, while the portions of the star farthest from the black hole form a tail of practically uniform thickness.

The density in the fluid tidally stripped off the star drops relatively quickly (within one orbital period for the bulk of the matter) below the threshold value ρ_0 , so that in effect the fluid condenses into a pressure-free fog, or dust moving on ballistic trajectories which are practically determined by the potential of the black hole. We see none of the expansion of stripped matter characteristic of a polytrope. This is simply because, in a sense, quark matter behaves more like a liquid than a gas—the volume of a mass m of quark fluid cannot exceed m/ρ_0 , a fact also evident in the behavior of the star itself, in which already in the initial stages the pressure decreases so quickly that the density *inside* the star drops below ρ_0 and cavitation occurs, see Fig. 1, especially panel e). This last result may not be robust: the initial central density in our Newtonian star is less than $3\rho_0$, so it takes less than a threefold increase of its volume for cavitation to occur. But when the equilibrium structure is governed by the relativistic TOV equation, the central density is closer to $5\rho_0$ (Witten, 1984), and a larger increase of volume can be accommodated.

Typically, a few high density ($\rho > \rho_0$) clumps break off the star or condense out of the accretion stream. Most of these are quickly accreted by the black hole. At the conclusion of tidal stripping a starlet of $\sim 10^{-2}M_\odot$ remains relatively far from the black hole, and on occasion is tidally injected into a highly elliptic orbit—such is the case in run C, where the $0.055M_\odot$ starlet visible in Figs. 2 and 3 is in a bound orbit and still moving away from the black hole with orbital speed of 5.4×10^4 km/s at the end of the simulation (see Fig. 4 for the center-of-mass velocity vector). The same tidal interaction has also substantially spun up the starlet to a rotational period of 1.3 ms (at the end of simulation C).

In the six runs presented here, the amount of mass ejected remains unresolved. At most, a few individual SPH particles (of mass M_{SS}/N each, see Table 1) are in unbound trajectories—the number of SPH particles ejected varies from none (zero) to three for the runs of Table 1. For instance, at the end of run C only one SPH particle (to the left of the $x = -200$ km tick-mark in Fig. 2, at $y \approx -390$ km) is on a clearly outbound trajectory, its terminal velocity (at infinity) will be about 45000 km/s, and its velocity is so high that it will not only leave the erstwhile binary, but the Galaxy and the Virgo cluster as well. The future of one more particle is undecided, it may or may not be bound to the starlet, whose fate is sealed. The starlet is doomed to undergo a close encounter with the black hole.

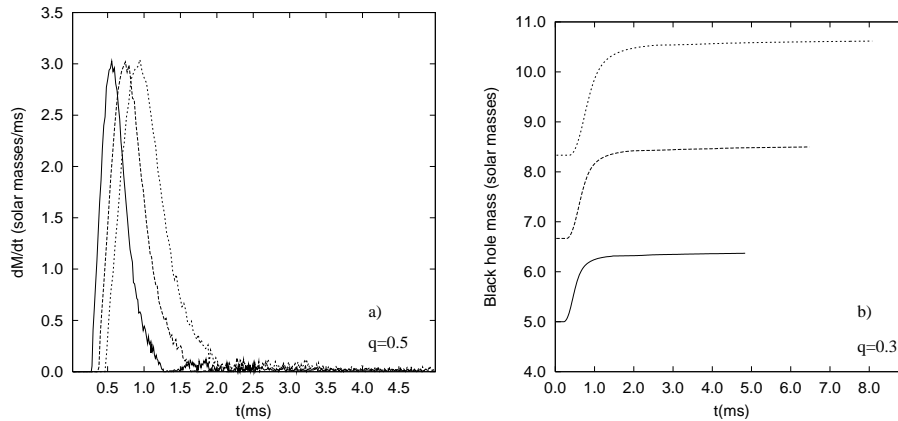


Figure 5: (a) Mass accretion rate onto the black hole as a function of time for runs A (solid line), C (dashed line) and E (dotted line). Accretion of individual “condensations” is clearly visible for run A starting at about 1.5 ms. (b) Black hole mass as a function of time for runs B (solid line), D (dashed line) and F (dotted line).

9 Discussion

It has long been noted that the tidal action of a black hole on a nearly incompressible star will stretch the latter into a long tube (Wheeler 1971). Mashhoon (1973) computes the shape of a uniform-density star approaching a Kerr black hole from infinity, and finds (neglecting accretion and forcing the star to be a tri-axial ellipsoid) that the star continues to be stretched even after passing the point of closest approach to the black hole (periholon?), at $1.961GM_{\text{BH}}/c^2$ for a particular choice of initial conditions, reaching a seven-fold elongation at a distance of $5.5GM_{\text{BH}}/c^2$. It is gratifying to note that our Newtonian simulation gives qualitatively similar results.

As in the case of stiff polytropes, we find that a small fragment of the star survives the first encounter and is placed into a higher orbit (papers I, III, Lee and Kluźniak 1995, Kluźniak and Lee 1998). In all cases (Table 1), we find that the mass of this quark starlet is $M_s = 0.03M_{\text{SS}}$. However, gravitational radiation will quickly lead to the coalescence with the black hole also of this starlet. The equilibrium structure of such low-mass quark stars is well described (at least up to the Jacobi turn-off) by Maclaurin spheroids (Amsterdamski et al. 2002), but at the end of the simulation the starlet is not yet in equilibrium. We will not speculate here on the astrophysical signatures of such an object, as we do not expect a similar result to hold for the Paczyński-Wiita potential. However, we do note that the starlet has been spun up by tidal interactions, this had not been

previously recognized.

We see no evidence of “figure 8” trajectories envisaged by Caldwell and Friedman (1991), and hence none of frequent and high-speed collisions between stellar fragments, but we do confirm their estimates of tidal fragmentation of the star to the limits of our resolution. With the exception of the starlet, the surviving fragments are definitely $< 10^{-2} M_{\odot}$ in mass, as the code would recognize any fragment of density exceeding ρ_0 and mass not less than $64 M_{SS}/N$ (each SPH particle has 64 “neighbors” in the current implementation); in fact, some even smaller high density fragments have been resolved, e.g., the soon-to-be-accreted blobs visible in panels g) and h) of Fig. 1. Paczyński (1991) suggests the formation of an accretion disk, and we do see a disk-like structure, but it is composed of small fragments of (quark) matter, rather like Saturn’s rings or Kuiper’s belt. At least, such is the case at the end of our simulation. Again, this is related to the existence of a minimum density, ρ_0 , of quark matter (Section 8), there simply is not enough fluid to fill the volume of a disk about 50 km in radius and a few kilometers thick (Figs. 2 and 3). Qualitatively, then, our simulation is in agreement with the relativistic calculations of Wheeler (1971) and Mashhoon (1973), and we find that some, but not all, of the expectations of Caldwell and Friedman (1991) and of Paczyński (1991) are supported by our study.

An unexpected result is the very rapid accretion of much of the material (Fig. 5). Most of the quark star has been devoured by the black hole in less than 2 ms. Inclusion of a pseudo-Newtonian potential will make this process even more drastic.

Finally, we find no definitive evidence of mass ejection from the system. With our resolution we place an upper bound of $< 3 \times 10^{-4} M_{\odot}$ on the amount of matter ejected, in all the cases considered. This is in stark contrast to the polytropic case (papers I, III), and is clearly a consequence of the equation of state of quark matter. Our limit on the mass ejected is lower than that allowed by the work of Lattimer and Schramm (1974) for neutron stars coalescing with black holes, but we have to agree with them that “the possibility of zero-mass ejection cannot be totally excluded.”

10 Conclusions

We have found no convincing evidence of ejection of quark matter from the binaries modeled. Further simulations, not reported here, show that inclusion of pseudo-Newtonian potentials only strengthens this conclusion. This may encourage other workers to continue studying the astrophysics of quark stars.

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12 References

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